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Register No. ☐☐☐☐☐☐

CHENNAI SCIENCE FORUM
PREPARATORY EXAMINATION

Time Allowed : 15 Min + 3 Hours]   **[Maximum Marks : 100**

PART – III   MATHEMATICS

Instructions : (1)   Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

(2)   Use Black or Blue ink to write and pencil to draw diagrams.

Note : This question paper contains **four** sections.

PART – I
(Marks : 14)   14 x 1 = 14

Note : (i)   Answer all the **14** questions.

(ii)   Choose the most suitable answer from the given four alternatives and write the option code with the corresponding answer.

1. If $A = \{a, b, p\}$, $B = \{2, 3\}$ and $C = \{p, q, r, s\}$ then $n\left[(A \cup C) \times B\right]$ is
   (a) 8   (b) 20   (c) 12   (d) 16

2. If $f(x) = 2x^2$ and $g(x) = \dfrac{1}{3x}$ then fog is
   (a) $\dfrac{3}{2x^2}$ (b) $\dfrac{2}{3x^2}$ (c) $\dfrac{2}{9x^2}$ (d) $\dfrac{1}{6x^2}$

3. Given $F_1 = 1$, $F_2 = 3$ and $F_n = F_{n-1} + F_{n-2}$ then F_5 is
   (a) 3   (b) 5   (c) 8   (d) 11

4. If a, b, c, l, m are in A.P then the value of $a - 4b + 6c - 4l + m$ is
   (a) 1   (b) 2   (c) 3   (d) 0

5. $\dfrac{3y-3}{y} \div \dfrac{7y-7}{3y^2}$ is
   (a) $\dfrac{9y}{7}$ (b) $\dfrac{9y^3}{21y-21}$
   (c) $\dfrac{21y^2-42y+21}{3y^3}$ (d) $\dfrac{7(y^2-2y+1)}{y^2}$

6. The GCD of $(x^3 + 1)$ and $x^4 - 1$ is
   (a) $x^3 - 1$ (b) $x^3 + 1$ (c) $x + 1$ (d) $x - 1$

7. If $\begin{pmatrix} 8 & 4 \\ x & 8 \end{pmatrix} = 4\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ then the value of x is
   (a) 1   (b) 2   (c) $\dfrac{1}{4}$ (d) 4

8. If in ΔABC, $DE \parallel BC$, $AB = 3.6cm$, $AC = 2.4cm$ and $AD = 2.1cm$ then the length of AE is
   (a) $1.4cm$ (b) $1.8cm$ (c) $1.2cm$ (d) $1.05cm$

9. The slope of the line which is perpendicular to a line joining the points $(0, 0)$ and $(-8, 8)$ is
   (a) -1 (b) 1   (c) 13   (d) -8

10. If $\sin \theta = \cos \theta$ then $2 \tan^2\theta + \sin^2\theta - 1$ is equal to
    (a) $\dfrac{-3}{2}$ (b) $\dfrac{3}{2}$ (c) $\dfrac{2}{3}$ (d) $\dfrac{-2}{3}$

11. A solid sphere of radius x cm is melted and cast into a shape of a solid cone of same radius. The height of the cone is
    (a) $3x\ cm$ (b) $x\ cm$ (c) $4x\ cm$ (d) $2x\ cm$

12. The C.S.A of a right circular cylinder of radius $1cm$ and height $1cm$ is equal to
    (a) $\pi\ cm^2$ (b) $2\pi\ cm^2$ (c) $3\pi\ cm^2$ (d) $2\ cm^2$

13. Variance of first 20 natural numbers is
    (a) 32.25   (b) 44.25   (c) 33.25   (d) 30

14. Probability of getting 3 heads or 3 tails in tossing a coin 3 times is
    (a) $\dfrac{1}{8}$ (b) $\dfrac{1}{4}$ (c) $\dfrac{3}{8}$ (d) $\dfrac{1}{2}$

PART – II
(Marks : 20)

Note : Answer only 10 questions.   **Question No. 28** is compulsory.   10 x 2 = 20

15. A relation 'f' is defined by $f(x) = x^2 - 2$, where $x \in \{-2, -1, 0, 3\}$
    (i) List the elements of f (ii) Is f a function ?

16. Let $f(x) = x^2 - 1$ Find $fofof$

17. Solve $3p^2 + 2\sqrt{5} p - 5 = 0$ by formula method.

18. Given $A = \begin{pmatrix} p & 0 \\ 0 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 0 & -q \\ 1 & 0 \end{pmatrix}$, $C = \begin{pmatrix} 2 & -2 \\ 2 & 2 \end{pmatrix}$ and if $BA = C^2$
    Find p and q

19. If a, b, c are in A.P show that $3^a, 3^b, 3^c$ are in G.P

20. How many terms of the series $1^3 + 2^3 + 3^3 + \dots + $ should be taken to get the sum 14400 ?

Turn over   **15**   – 2 –

21. ΔLMN is a right angled triangle with ∠L = 90° A circle is inscribed in it. The lengths of the sides containing the right angle are $6cm$ and $8cm$. Find the radius of the circle.

22. If the points $P(-1, -4)$, $Q(b, c)$ and $R(5, -1)$ are collinear and if $2b + c = 4$, then find the values of b and c

23. Find the equation of a straight line passing through the points $(-3, 2)$ and $(4, 5)$

24. Prove that $\sqrt{\dfrac{1+\sin\theta}{1-\sin\theta}} = \sec\theta + \tan\theta$

25. A garden roller whose length is $3m$ long and whose diameter is $2.8\,m$ is rolled to level a garden. How much area will it cover in 8 revolutions ?

26. A hemispherical hollow bowl has material of volume $\dfrac{436}{3}\pi$ $cu.cm.$ Its external diameter is $14cm$. Find its thickness.

27. Find the standard deviation of first 12 natural numbers.

28. A die is rolled and a coin is tossed simultaneously. Find the probability that the die shows an odd number and the coin shows a head.

PART – III
(Marks : 50)

Note : Answer only 10 questions. **Question No. 42** is compulsory.　　　10 x 5 = 50

29. Find the sum to n terms of the series $0.4 + 0.44 + 0.444 +...$ to n terms.

30. The sum of first n, $2n$, and $3n$ terms of an A.P are S_1, S_2 and S_3 respectively. Prove that $S_3 = 3(S_2 - S_1)$

31. If the function $f : R \to R$ is defined by $f(x) = \begin{cases} 2x+7, & x < -2 \\ x^2 - 2, & -2 \le x < 3 \\ 3x - 2, & x \ge 3 \end{cases}$

    Find the values of　(i) $f(4)$　　(ii) $f(-2)$　　(iii) $\dfrac{f(1) - 3f(4)}{f(-3)}$

32. Let $A = \{x \in W \, / \, x < 2\}$, $B = \{x \in N \, / \, 1 < x \le 4\}$ and $C = \{3, 5\}$ verify that $A \times (B \cap C) = (A \times B) \cap (A \times C)$

33. If α, β are the roots of the equation $x^2 + 6x - 4 = 0$ then find the quadratic equation whose roots are α^2 and β^2

34. State and prove the "Alternate Segment Theorem"

35. If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ show that $A^2 - (a + d)A = (bc - ad)I$

36. Let $A(3, -4)$, $B(9, -4)$, $C(5, -7)$ and $D(7, -7)$
    Show that ABCD is a trapezium.

37. From the top of a $12m$ high building the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 30°. Determine the height of the tower.

38. A vessel is in the form of hemispherical bowl mounted by a hollow cylinder. The diameter is $14cm$ and the height of the vessel is $13cm$. Find the capacity of the vessel.

39. A right circular cylindrical container of base radius $6cm$ and height $15cm$ is full of ice cream. The ice cream is to be filled in cones of height $9cm$ and base radius $3cm$ having a hemispherical cap. Find the number of cones needed to empty the container.

40. Find the standard deviation for the following data:

x	45	50	55	60	65	70
y	5	13	4	9	5	4

41. Two dice are rolled together. Find the probability of getting a doublet or sum of faces as 4

42. Find the equation of a straight line passing through $(1, -4)$ and has intercepts which are in the ratio 2 : 5

PART – IV
(Marks : 16)

Note : Answer the following questions :　　　　　　2 x 8 = 16

43. (a) Draw the graph of $y = x^2 + 3x + 2$ and use it to solve $x^2 + 2x + 1 = 0$

    OR

    (b) Solve : $\dfrac{x}{2} - 1 = \dfrac{y}{6} + 1 = \dfrac{z}{7} + 2$, $\dfrac{y}{3} + \dfrac{z}{2} = 13$

44. (a) Construct a triangle ABC of base BC = 5.6 cm, ∠A = 40° and the bisector of ∠A meets BC at D such that CD = 4 cm

    OR

    (b) Draw a circle of radius 3 cm. Take a point P on this circle and draw a tangent at P

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