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CHENNAI SCIENCE FORUM

PREPARATORY EXAMINATION

PART - III MATHEMATICS

Time Allowed :15 Min + 3 Hours] [Maximum Marks : 90

Instructions : (1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

(2) Use Black or Blue ink to write and pencil to draw diagrams.

PART – I 20 x 1 = 20

Note : (i) All the questions are compulsory.
(ii) Choose the most suitable answer from the given **four** alternatives and write the option code and corresponding answer.

1. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 1 - |x|$ then the range of f is
(a) $(1, \infty)$ (b) $(-\infty, 1)$ (c) $\mathbb{R}$ (d) $(-1, \infty)$

2. The number of solutions of $x^2 + |x - 1| = 1$ is
(a) 2 (b) 3 (c) 0 (d) 1

3. $\cos1^\circ + \cos2^\circ + \cos3^\circ + \ldots + \cos179^\circ =$
(a) –1 (b) 1 (c) 89 (d) 0

4. Number of sides of a polygon having 44 diagonals is
(a) 11 (b) 22 (c) 4 (d) 4!

5. The coefficient of x^3 in the series e^{-2x} is
(a) $\frac{2}{3}$ (b) $\frac{4}{15}$ (c) $\frac{3}{2}$ (d) $\frac{-4}{15}$

6. The image of the point (2, 3) in the line $y = -x$ is
(a) (3, 2) (b) (–2, –3) (c) (–3, –2) (d) (–3, 2)

7. If the square of the matrix $\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is the unit matrix of order 2 then α, β and γ should satisfy the relation.
(a) $1 + \alpha^2 + \beta\gamma = 0$ (b) $1 - \alpha^2 + \beta\gamma = 0$ (c) $1 - \alpha^2 - \beta\gamma = 0$ (d) $1 + \alpha^2 - \beta\gamma = 0$

8. The value of $\theta \in \left[0, \frac{\pi}{2}\right]$ for which the vectors $\vec{a} = (\sin\theta)\hat{i} + (\cos\theta)\hat{j}$ and $\vec{b} = \hat{i} - \sqrt{3}\hat{j} + 2\hat{k}$ are perpendicular is equal to
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

[Turn Over

9. $\lim_{n\to\infty}\left[\frac{1}{n^2} + \frac{2}{n^2} + \frac{3}{n^2} + \ldots\ldots + \frac{n}{n^2}\right]$ is
(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) ∞

10. If $y = mx + c$ and $f(0) = f'(0) = 1$, then $f(2)$ is
(a) 2 (b) 1 (c) –3 (d) 3

11. $\int\sqrt{\frac{1-x}{1+x}}.dx$ is
(a) $\sin^{-1}x - \sqrt{1-x^2} + c$ (b) $\sqrt{1-x^2} + \sin^{-1}x + c$
(c) $\log\left|x + \sqrt{1-x^2}\right| - \sqrt{1-x^2} + c$ (d) $\sqrt{1-x^2} + \log\left|x + \sqrt{1-x^2}\right| + c$

12. It is given that the events A and B are such that $P(A) = \frac{1}{4}$, $P(A/B) = \frac{1}{2}$ and $P(B/A) = \frac{2}{3}$ then $P(B)$ is (a) $\frac{2}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{6}$

13. If the function $f : C \to C$ be defined by $f(x) = x^2 - 1$, then $f^{-1}(7)$ is
(a) $\pm\sqrt{5}$ (b) $\pm5\sqrt{2}$ (c) $\pm2\sqrt{2}$ (d) ± 6

14. If $i^2 = -1$, then $\begin{vmatrix} 2+3i & 3+2i \\ -3+2i & 2-3i \end{vmatrix}$ is
(a) 26 (b) 0 (c) $-4 + 9i$ (d) 20

15. If $|x| < 1$ and $y = 1 + x + x^2 + \ldots\ldots$ to ∞, then the value of $\frac{dy}{dx}$ is
(a) $1 + 2x + 3x^2$ (b) $\frac{-1}{(1+x)^2}$ (c) $\frac{-1}{(1-x)^2}$ (d) $\frac{1}{(1-x)^2}$

16. The value of $\int\frac{1}{x + x\log x}dx$ is
(a) $\log(1 + \log x)$ (b) $1 + \log x$ (c) $x + \log x$ (d) $x\log(1 + \log x)$

17. If $\vec{b}$ is a unit vector such that $\left(\vec{a} + \vec{b}\right).\left(\vec{a} - \vec{b}\right) = 8$, then $|\vec{a}|$ is
(a) ±3 (b) $\pm2\sqrt{2}$ (c) 1 (d) 3

18. If A, B are independent events such that P(A) = 0.3, $P(A \cup B) = 0.5$, $P\left(\frac{A}{B}\right) - P\left(\frac{B}{A}\right)$ is (a) $\frac{2}{7}$ (b) $\frac{3}{35}$ (c) $\frac{1}{70}$ (d) $\frac{1}{7}$

19. The value of $\frac{(\cos20^\circ + \sin20^\circ)}{(\cos20^\circ - \sin20^\circ)}$ is (a) $\tan75^\circ$ (b) $\cot25^\circ$ (c) 1 (d) $\cot65^\circ$

20. Find the odd one out of the following:
(a) $x^4 + 4x^3 + 12x^2$ (b) $(2x^2 + 4x - 3)(3x^2 - 5x + 2)$
(c) $x^2 - 2x + 2$ (d) $(x^2 + 1)(x - 3)(x + 4)$

PART – II 7 x 2 = 14

Note : (i) Answer any seven questions.
(ii) Question No.30 is compulsory and choose any six from the remaining.

21. Find the largest possible domain for the real valued function f is defined by $f(x) = \sqrt{x^2 - 5x + 6}$

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22. Evaluate : $\left(\left((256)^{\frac{-1}{2}}\right)^{\frac{1}{4}}\right)^{3}$

23. Solve: $\tan 2x = -\cot\left(x+\frac{\pi}{3}\right)$

24. Find the sum : $1+\frac{4}{5}+\frac{7}{25}+\frac{10}{125}+\ldots\ldots\ldots$

25. If θ is a parameter, find the equation of the locus of a moving point, whose coordinates are $x = a\cos^3\theta$, $y = a\sin^3\theta$

26. If A is a square matrix such that $A^2 = A$. Find the value of $(A-I)^3 + (A+I)^3 - 7A$

27. Find the relation between a and b if $\lim_{x\to 3} f(x)$ exists where

$$f(x) = \begin{cases} ax + b & ; x > 3 \\ 3ax - 4b + 1 & ; x < 3 \end{cases}$$

28. Find the slope of the tangent to the curve $xy = c^2$ at $(ct, c/t)$

29. Evaluate : $\int e^x(\tan x + \log\sec x).\,dx$

30. If $\vec{a}\times\vec{b} = \vec{c}\times\vec{d}$, $\vec{a}\times\vec{c} = \vec{b}\times\vec{d}$ then prove that $(\vec{a}-\vec{d})$ is parallel to $(\vec{b}-\vec{c})$

PART – III

Note : (i) Answer any **seven** questions. 7 x 3 = 21

(ii) Question No. **39** is compulsory and chooses any six from the remaining.

31. Solve : $\log_8^x + \log_4^x + \log_2^x = 11$

32. A plane is $1km$ from one landmark and $2km$ from another. From the plane point of view the land between them. Substends an angle of 45°. How far apart the landmarks ?

33. Find the number of strings that can be made using all letters of the word THING. If these words are written as in a dictionary, what will be the 85th string.

34. Prove that $\sqrt{\frac{1-x}{1+x}}$ is approximately equal to $1 - x + \frac{x^2}{2}$ when x is very small.

35. The slope of one of the straight lines $ax^2 + 2hxy + by^2 = 0$ is three times the other, show that $3h^2 = 4ab$

36. If $\vec{a}, \vec{b}, \vec{c}$ are position vectors of the vertices A, B, C of a triangle ABC, show that the area of the triangle ABC is $\frac{1}{2}\left|\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a}\right|$ Also deduce the condition for collinearity of the points A, B and C

37. Do the limits of following function exist as $x \to 0$?

State the reason for your answer $\frac{\sin(x-|x|)}{x-|x|}$

38. If $y = \sqrt{x+\sqrt{x}}$ find $\frac{dy}{dx}$

39. Evaluate : $\int x.\log x.\,dx$

[Turn Over

40. Find the value of $\begin{vmatrix} \log_3^{512} & \log_4^3 \\ \log_3^8 & \log_4^9 \end{vmatrix} \times \begin{vmatrix} \log_2^3 & \log_8^3 \\ \log_3^4 & \log_3^4 \end{vmatrix}$

PART – IV

Note : Answer ***all*** the questions. 7 x 5 = 35

41. (a) If $f: \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = 3x - 5$, prove that f is a bijection and find its inverse. **OR**

(b) Show that: $\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = (a+b+c)(a-b)(b-c)(c-a)$

by using factor theorem.

42. (a) Solve the rational expressions into partial fraction $\frac{(x-1)^2}{x^3+x}$ **OR**

(b) Show that the points whose position vectors $\hat{4i} + \hat{5j} + \hat{k}$, $-\hat{j} - \hat{k}$, $3\hat{i} + 9\hat{j} + 4\hat{k}$ and $-\hat{4i} + \hat{4j} + \hat{4k}$ are coplanar.

43. (a) If A + B + C = 180°, then prove that :

$\sin^2 A + \sin^2 B - \sin^2 C = 2\sin A \sin B \cos C$ **OR**

(b) Evaluate : $\lim_{x\to\pi/4} \frac{4\sqrt{2} - (\cos x + \sin x)^5}{1 - \sin 2x}$

44. (a) Using Mathematical Induction, show that for any natural number n, with the assumption $i^2 = 1$. $\left(r(\cos\theta + i\sin\theta)\right)^n = r^n(\cos n\theta + i\sin n\theta)$ **OR**

(b) If $y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$, show that $(1-x^2)y_2 - 3xy_1 - y = 0$

45. (a) Prove that $\sqrt[3]{x^3+7} - \sqrt[3]{x^3+4}$ is approximately equal to $1/x^2$ when x is large. **OR**

(b) Evaluate : $\int \frac{2x+3}{\sqrt{x^2+x+1}}.dx$

46. (a) Show that the equation $9x^2 - 24xy + 16y^2 - 12x + 16y - 12 = 0$ represents a pair of parallel lines. Find the distance between them. **OR**

(b) The chances of A, B and C becoming manager of certain company are 5 : 3 : 2. The probabilities that the office canteen will be improved if A, B and C become managers are 0.4, 0.5, 0.3 respectively. If the office canteen has been improved, what is the probability that B was appointed as the manager ?

47. (a) Without expanding the determinant, Prove that

$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$ **OR**

(b) Differentiate : $\tan^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$ with respect to $\cos^{-1}(x^2)$

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