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CHENNAI SCIENCE FORUM

PREPARATORY EXAMINATION

PART – III MATHEMATICS

Time Allowed: 15 Min + 3 Hrs] **[Maximum Marks : 90**

Instructions : (1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

(2) Use Black or Blue ink to write and pencil to draw diagrams.

PART – I 20 x 1 = 20

Note : (i) All the questions are compulsory.

(ii) Choose the most suitable answer from the given **four** alternatives and write the option code and corresponding answer.

1. If A is a non-singular matrix such that $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$, then $(A^T)^{-1} =$

(1) $\begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ (3) $\begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$ (4) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$

2. If α and β are the roots of $x^2 + x + 1 = 0$, then $\alpha^{2020} + \beta^{2020}$ is

(1) –2 (2) –1 (3) 1 (4) 2

3. The polynomial $x^3 + 2x + 3$ has

(1) one negative and two imaginary zeros
(2) one positive and two imaginary zeros
(3) three real zeros (4) no zeros

4. $\sin^{-1}(2\cos^2 x - 1) + \cos^{-1}(1 - 2\sin^2 x) =$

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{6}$

5. If $x + y = k$ is a normal to the parabola $y^2 = 12x$, then the value of k is

(1) 3 (2) –1 (3) 1 (4) 9

6. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar unit vectors such that

$\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\vec{b} + \vec{c}}{\sqrt{2}}$ then the angle between $\vec{a}$ and $\vec{b}$ is

(1) $\frac{\pi}{2}$ (2) $\frac{3\pi}{4}$ (3) $\frac{\pi}{4}$ (4) π

7. What is the value of the limit $\lim_{x \to 0}\left(\cot x - \frac{1}{x}\right)$?

(1) 0 (2) 1 (3) 2 (4) ∞

8. If $f(x,y,z) = xy + yz + zx$, then $f_x - f_z$ is equal to

(1) $z - x$ (2) $y - z$ (3) $x - z$ (4) $y - x$

9. The value of $\int_0^1 x(1-x)^{99}dx$ is

(1) $\frac{1}{11000}$ (2) $\frac{1}{10100}$ (3) $\frac{1}{10010}$ (4) $\frac{1}{10001}$

10. If sin x is the integrating factor of the linear differential equation $\frac{dy}{dx} + Py = Q$, then P is

(1) log sin x (2) cos x (3) tan x (4) cot x

11. Let X have a Bernoulli distribution with mean 0.4, then the variance of $(2X - 3)$ is

(1) 0. 24 (2) 0. 48 (3) 0. 6 (4) 0. 96

12. The proposition $p \wedge (\neg p \vee q)$ is

(1) a tautology (2) a contradiction
(3) logically equivalent to $p \wedge q$ (4) logically equivalent to $p \vee q$

13. The order and degree of the differential equation $(y'')^2 + y' = x(x + y'')^2$ is

(1) 2, 1 (2) 2, 2 (3) 1, 1 (4) 1, 2

14. If $\sin\left[\sin^{-1}\left(\frac{2}{5}\right) + \cos^{-1} x\right] = 1$, then the value of x is

(1) $\frac{1}{5}$ (2) $\frac{2}{5}$ (3) 0 (4) $\frac{\pi}{2}$

15. The locus of the point of intersection of perpendicular tangents to the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ is

(1) $x^2 + y^2 = 25$ (2) $x^2 + y^2 = 34$ (3) $x^2 - y^2 = 16$ (4) $x^2 - y^2 = 16$

16. If $a * b = \frac{ab}{3}$, $a, b \in Q^+$, then the inverse of "18" is

(1) 18 (2) $\frac{1}{18}$ (3) $\frac{1}{2}$ (4) none of these

17. If the system of equations $x = cy + bz$, $y = az + cx$ and $z = bx + ay$ has a non-trivial solution, then the value of $a^2 + b^2 + c^2 + 2abc$ is

(1) 0 (2) –1 (3) 1 (4) $\frac{1}{2}$

18. If $y = 7x - x^3$ and x increases at a rate of 4 units per second, how fast is the slope of the curve changing when $x = 1$?

(1) – 48 (2) – 24 (3) 48 (4) 24

19. If $\hat{a}, \hat{b}, \hat{c}$ are three unit vectors such that $\hat{b}, \hat{c}$ are non-parallel and $\hat{a} \times (\hat{b} \times \hat{c}) = \frac{1}{2}\hat{b}$, then the angle between $\hat{a}$ and $\hat{b}$ is

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{2}$ (4) 0°

20. If $\sqrt{x + iy} = 2 + 3i$, then $x + y$ is

(1) –7 (2) 5 (3) 7 (4) none of these

PART – II

Note : (i) Answer any seven questions. 7 x 2 = 14
(ii) Question No.**30** is compulsory and choose any six from the remaining.

21. Find the inverse of the non-singular matrix $A = \begin{bmatrix} 0 & 5 \\ -1 & 6 \end{bmatrix}$, by Gauss-Jordan method.
22. Simplify : $\sec^{-1}\left(\sec\left(\frac{5\pi}{3}\right)\right)$
23. Obtain the equation of the circle for which (3, 4) and (2, –7) are the ends of a diameter.
24. State and Prove Jacobi's Identity.
25. Explain why Lagrange's mean value theorem is not applicable to the following functions in the respective intervals: $f(x) = |3x+1|,\ x \in [-1, 3]$
26. If $v(x, y) = x^2 - xy + \frac{1}{4}y^2 + 7,\ \ x, y \in R$ find the differential dv
27. Find the area of the region bounded by x-axis, the curve $y = \cos x$, the lines $x = 0$ and $x = \pi$
28. The mean and variance of a binomial variate X are respectively 2 and 1.5. Find $P(X \geq 1)$
29. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ be any two Boolean matrices of the same type. Find $A \vee B$ and $A \wedge B$
30. If α and β are the complex cube roots of unity. Prove that $\alpha^4 + \beta^4 + \alpha^{-1}.\beta^{-1} = 0$

PART – III

Note : (i) Answer any seven questions. 7 x 3 = 21
(ii) Question No. **40** is compulsory and choose any six from the remaining.

31. Given $A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$, Find a matrix X such that A X B = C
32. If $\frac{1+z}{1-z} = \cos 2\theta + i\sin 2\theta$, show that $z = i\tan\theta$
33. If p and q are roots of the equation $lx^2 + nx + n = 0$ show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$
34. Prove that the point of intersection of the tangents at 't_1' and 't_2' on the parabola $y^2 = 4ax$ is $[a t_1 t_2,\ a(t_1 + t_2)]$
35. Find the foot of the perpendicular drawn from the point (5 , 4 , 2) to the line $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1}$ Also, find the equation of the perpendicular.
36. Find the local extrema for the following function using second derivative test: $f(x) = x \log x$
37. Assuming $\log_{10} e = 0.4343$, find an approximate value of $\log_{10} 1003$
38. Solve the following Linear differential equation : $\frac{dy}{dx} + \frac{y}{x\log x} = \frac{\sin 2x}{\log x}$
39. In a binomial distribution consisting of 5 independent trials, the probability of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the mean and variance of the distribution.
40. Evaluate : $\int_{1/e}^{e} |\log x|\, dx$

PART – IV

Answer ALL the questions: 7 x 5 = 35

41.(a) The upward speed $v(t)$ of a rocket at time t is approximated by $v(t) = at^2 + bt + c,\ 0 \leq t \leq 100$ where a, b, and c are constants. It has been found that the speed at times $t = 3$, $t = 6$, and $t = 9$ seconds are respectively, 64, 133 and 208 miles per second respectively. Find the speed at time $t = 15$ seconds. (Use Gaussian Elimination Method) **OR**

(b) Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity and (iv) existence of inverse for the operation X_{11} on a subset A = { 1, 3, 4, 5, 9 } of the set of remainders { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 }

42.(a) Solve the equation $z^3 + 8i = 0$, where $z \in \mathbb{C}$ **OR**

(b) If the probability that a fluorescent light has a useful life of at least 600 hours is 0.9, find the probabilities that among 12 such lights.
(i) exactly 10 will have a useful life of at least 600 hours;
(ii) at least 11 will have a useful life of at least 600 hours;
(iii) at least 2 will not have a useful life of at least 600 hours.

43. (a) If $2 + i$ and $3 - \sqrt{2}$ are roots of equation $x^6 - 13x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 = 0$, find all roots. **OR**

(b) A pot of boiling water at 100°C is removed from a stove at time t = 0 and left to cool in the kitchen. After 5 minutes, the water temperature has decreased to 80°C, and another 5 minutes later it has dropped to 65°C. Determine the temperature of the kitchen.

44.(a) If $a_1, a_2, a_3, \ldots\ldots, a_n$ is an arithmetic progression with common difference d, prove that

$$\tan\left[\tan^{-1}\left(\frac{d}{1+a_1 a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_1 a_3}\right) + \ldots + \tan^{-1}\left(\frac{d}{1+a_n a_{n-1}}\right)\right] = \frac{a_n - a_1}{1 + a_1 a_n}$$ **OR**

(b) Using integration find the area of the region bounded by triangle ABC, whose vertices A, B and C are (–1, 1), (3, 2) and (0, 5) respectively.

45.(a) Identify the type of conic and find centre, foci, vertices and directrices of : $9x^2 - y^2 - 36x - 6y + 18 = 0$ **OR**

(b) $W(x, y, z) = xy + yz + zx,\ \ x = u - v,\ \ y = uv,\ \ z = u + v,\ \ u, v \in \mathbb{R}$ Find $\frac{\partial W}{\partial u}, \frac{\partial W}{\partial v}$ and evaluate them at $\left(\frac{1}{2}, 1\right)$

46.(a) Find the non-parametric form of vector equation and Cartesian equation of the plane passing through the point (1, –2, 4) and perpendicular to the plane $x + 2y - 3z = 11$ and parallel to the line $\frac{x+7}{3} = \frac{y+3}{-1} = \frac{z}{1}$ **OR**

(b) Sketch the graph of the function $y = \frac{3x}{x^2 - 1}$

47.(a) If the curves $y^2 = 4ax$ and $xy = c^2$ cut orthogonally, prove that $c^4 = 32a^4$ **OR**

(b) Find the volume of the loop of the curve: $x = t^2;\ y = t - \frac{t^3}{3}$ about the x-axis.

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